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ON THE PROBLEM OF CONTROL OF RELAXATION OSCILLATIONS OF A "SINGING" FLAME

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Possible solutions of the problem of control of constant-amplitude relaxation oscillations of a "singing" flame are considered using mathematical modeling. Methods for reducing the amplitude of such oscillations are proposed.

Nonstationary combustion in different devices is used in many fields of technology. It is common knowledge that control of thermoacoustic oscillations and monitoring of them are quite limited even in the simplest cases, since the reasons and conditions for their occurrence and the mechanisms of sustaining them remain to be elucidated [1]. This nonstationary phenomenon is also responsible for explosions in mines and for vibrational combustion in the combustion chambers of jet engines and primary furnaces of industrial units.

Substantiation of the impetus to solving various questions relating to this problem and an analysis of publications and experimental and theoretical investigations performed have been presented in numerous monographs.

Different hypotheses for the reasons why periodic pulsations in vertical primary furnaces occur have been expressed [2] and different methods for damping them have accordingly been proposed. According to [3], vibrational combustion is both combatted and realized blindly and with considerable testing expenses [4, 5]. The problem of reduction in the oscillation amplitude is sometimes solved by pure accident [4, 5]. Also, it is noteworthy that "combustion science is on the threshold of a new flight" [6], and the foundation for solution of the vibrational-combustion problem has been laid in [3], where the most significant features of this nonstationary regime have thoroughly been analyzed and evaluated. It is noted in the above monograph that the energy sources for sustaining vibrational-combustion oscillations are the potential pressure energy growing in heat supply (according to Rayleigh) and the kinetic flow energy (according to B. V. Raushenbakh).

The mechanical rotational energy in the grid of the compressor impeller is converted to the head $H(Q)$ [7] and to the internal energy of the medium because of the increase in the pressure and temperature in accordance with the process of its compression. In combustion of fuel, the energy released is converted in the reverse order: first to the internal energy and, as it grows, to the components of the head. On this basis, we have introduced the head characteristic $H(Q)$ of heat supply in [8]; this characteristic represents the dependence of the pressure of the resultant of all forces in the oscillatory circuit of the flow, distributed over the cross-sectional area, on the flow rate. Then we replaced the energy equation in the system of gas-mechanics equations by the head characteristic $H(Q)$ of heat supply. Thus, the problem of theoretical description of self-oscillations resulting from the heat-to-head conversion was reduced to the problem of surging of a rotary (vane) compressor [7] whose head characteristic is determined experimentally.

The equations that describe motions corresponding to the phenomena in question and to those that are stationary are formally coincident. This is due to the fact that they are based on the general laws of conservation of momentum and mass and on the use of the polytropic equation [7].

For the solution of the system of such equations we preserved the algorithm adopted in [7], but for its computer-aided realization we modified the Eulerian method of construction of integral curves [8] and developed a method for determining periodic solutions of differential equations with a retarded argument [9]. Such a theoretical description of heat-supply-induced oscillations was, apparently, made by the author for the first time, which formed the basis for developing and updating thermoacoustic generators [10].

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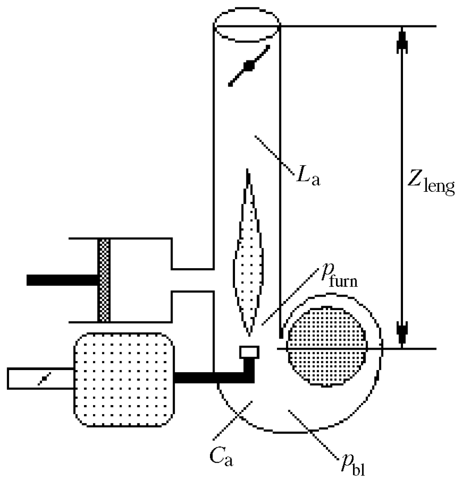

Fig. 1. Model of a Riecke tube with a "singing" flame and forced air feed by a vane blower.

Formulation of the Problem and Substantiation of Its Solution. Motion in a vertical tube in which a "singing" flame occurs [1] may be natural (Riecke tube) or be additionally created by a fan (Fig. 1). The reason for the excitation of self-oscillations due to heat supply or their necessary condition is the presence of the ascending branch on the head characteristic $H(Q, W)$ of the Riecke tube with an electric coil [8] and on the characteristic of heat supply $H(Q)$ of the singing flame. Furthermore, the mode and parameters of oscillations in the Riecke tube are dependent, in a certain manner, on the phenomenological retardation of combustion τ and on the distinctive features of "turbulent" combustion (in terms of [5]) where the electric coil is replaced by a gas burner burning gas.

With decrease in the wave resistance Z of the primary furnace, when the condition $Z < Z^*$ is fulfilled [11], thermoacoustic oscillations become relaxation ones with a constant amplitude and further reduction in Z values. A regularity of such alternations in the flow parameters is growth in their amplitude with heat-supply power W [8], which is also observed in the furnaces of stoves of blast furnaces in which the dynamic load on the structure becomes destructive [12].

Since increase in the heat load and hence in the blowing temperature yields a multimillion economic effect only on one blast furnace [12] due to the decrease in the burden coke, the problem of reduction in the oscillation amplitude of vibrational combustion is pressing. Below, we consider the solution of such a problem using mathematical modeling for relaxation self-oscillations of vibrational combustion by acting on the mechanisms of their sustaining and by possible changing the wave resistance of the oscillatory circuit. The efficiency of solution of this problem is illustrated by oscillations in a Riecke tube (Fig. 1) with forced motion created by a fan.

Nonstationary motions in the device (Fig. 1) with the manifestation of a phenomenological combustion retardation τ [13] are described by the system of equations [8, 10] with a retarded argument

$$L_a \frac{dQ_T}{dt} = H(Q_T) - P ,$$
$$C_a \frac{dP}{dt} = Q_T - \varphi\,[P(t-\tau)] , \qquad (1)$$

where $P = p_{bl} - p_{furn}$, $Q_{in}(t-\tau) = \varphi[P(t-\tau)]$, and the inversion of this function [7] is the quadratic dependence of the head loss on the burner and on the regulator (if it is built into the structure) on the flow rate $Q_{in}(t-\tau)$. The character of the head characteristic of the oscillatory circuit (Fig. 1) $H(Q_T) = H_f(Q_T) + A(Q_T) - h_{leng}(Q_T) - h_t(Q_T)$ is determined by the dependences characterizing the head supply to the flow because of the corresponding conversion of the mechanical rotational energy to the head in the impeller grid $H_f(Q_T)$ and conversion of the supplied heat $A(Q_T)$ and by the dependences of the head loss along the pipeline length $h_{leng}(Q_T)$ and of that caused by the heat supply $h_t(Q_T)$ [8–11]; these dependences are determined by calculation, except for the characteristics of the blower $H_f(Q_T)$.

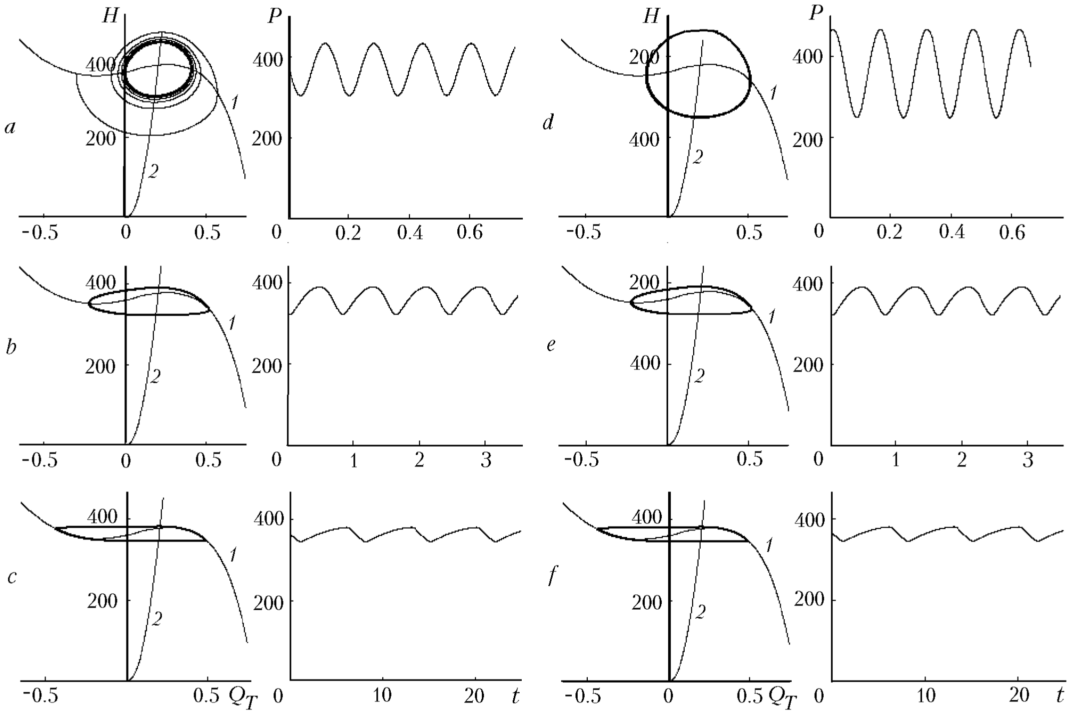

Fig. 2. Transformation of the limit cycle of harmonic oscillations to a limit cycle of relaxation oscillations of variable and constant amplitudes with decrease in the wave resistance: a and d) Z = 325.09, b and e) 79.63, and c and f) 17.8 Pa·sec/m^3; a–c) τ = 0 and d–f) 0.04 sec (1) $H(Q_T)$; 2) $h_n = k_n(Q_T^2)$. P and H, Pa; Q_T, m^3/sec.

The dependence $H(Q_T)$ for different industrial arrangements of primary furnaces is described, on the whole, by the polynomial of the third or fifth degree. The head characteristics of vane blowers are of the same nature [7], since they are determined by the theoretical characteristic of conversion of just the mechanical energy to the head and by the hydraulic friction and vortex-formation loss subtracted from it.

The system of equations (1) is formally coincident with the equations of self-oscillation (surging) theory; therefore, in constructing its periodic solutions, the initial parameters are determined, according to [7], by the stationary regime selected, and their deviations are prescribed arbitrarily, since self-oscillations and limit cycles (Fig. 2) are independent of them. By the mechanisms contributing to the excitation of vibrations and not related to the presence of the retardation τ, we will mean factors giving rise to the ascending branch of the characteristic $H(Q_T)$, when we have τ = 0 in the system of equations (1).

To eliminate the influence of surging due to the character of energy supplied by the fan on the nonstationary motions because of the heat supply and to simplify the problem we take its head characteristic $H_f(Q_T)$, for the flow rates fed to the system, such that the head remains constant with flow rate, i.e., we have $dH_f/dQ_T = 0$. In the region of negative flow rates, the characteristic $H_f(Q_T)$ is supplemented with the corresponding branch of the parabola.

A necessary condition of periodic solution of system (1) for τ = 0 is the presence of the ascending branch on the $H(Q_T)$ plot. The sufficient condition is determined by the value of the wave resistance Z of the oscillatory circuit, for which a limit cycle (i.e., an isolated closed solution) of the equation of integral curves of system (1) (just as in [7], the equation is obtained by eliminating the time t from the system) is formed:

$$\frac{dP}{dQ_T} = \frac{Q_T - \varphi(P)}{H(Q_T) - P} \frac{L_a}{C_a} . \tag{2}$$

The wave resistance of the oscillatory circuit $Z = \sqrt{L_a/C_a}$ in the absence of lumped volumes is represented in the form [14] $Z = \rho_T a_T / S$.

If the value of the phenomenological retardation is $\tau \neq 0$, formal division of the equations of system (1) by each other leads to Eq. (2) with a retarded argument $t - \tau$ and an explicit independence from the time t. Next, assuming an arbitrary initial function [9] on the retardation segment $0 \le t \le \tau$, we obtain a recurrence series of ordinary differential equations with no explicit dependence on time; the solutions of these equations form the limit cycle of the initial equation with retardation.

A characteristic feature of the process of combustion in the device in question is that it becomes possible to simultaneously increase both the temperature and the flow rate of smoke with increase in the heat load. Therefore, along with the well-known mechanisms of oscillation excitation and sustaining due to the phenomenological retardations of the process of combustion [13], heat transfer at high temperature gradients [15], and volumetric relaxation [1], we have other mechanisms that contribute to the formation of the ascending branch on the characteristic $H(Q_T)$.

Mechanisms of Thermoacoustic Oscillations for a Heat-Flux Power Varying with Flow Rate. The presence of the descending branches on the $h_{\text{leng}}(Q_T)$ or $h_{\text{t}}(Q_T)$ plots and the growing character of change in the lift pressure $A(Q_T) = gZ_{\text{leng}}(\rho_0 - \rho_T)$ contribute to the formation of the ascending branch of the head characteristic $H(Q_T)$.

To find the conditions of appearance of the $h_{\text{t}}(Q_T)$ descending branch we take that the hydraulic head loss $h_{\text{leng}}(Q_T)$ is absent, i.e., the compressible fluid is hydraulically ideal. Since the flow head decreases with growth in $h_{\text{t}}(Q_T)$ and increases with its reduction, we determine the character of variation in the dependence $h_{\text{t}}(Q_T)$ from the change in the head $H(Q_T)$ in heat supply. For this purpose we use the equation of the first law of thermodynamics in differential form for flow in a horizontal channel

$$dq = du + d\left(\frac{P}{\rho_T}\right) + d\frac{c^2}{2} \tag{3}$$

or

$$dH(Q_T) = \rho_T d(q - u) . \tag{4}$$

Since the heat supplied in the polytropic process is $dq = c_v \frac{n-k}{n-1} dT$, and the change in the internal energy is $du = c_v dT$, with allowance for the dependence of the temperature on the volumetric flow rate $T = T_0 + \psi(Q_T)$ we may write Eq. (4) as

$$dH(Q_T) = \rho_T c_v \frac{1-k}{n-1} d\psi(Q_T) . \tag{5}$$

If the function $\psi(Q_T)$ is increasing, the ascending branch of the function $H(Q_T)$ results from the descending branch of $h_{\text{t}}(Q_T)$ and, according to Eq. (5), occurs in the polytropic process of heat supply with an exponent of the process n for which $0 \le n < 1$, which is realized in vertical primary furnaces.

In the case where the function $\psi(Q_T)$ is decreasing, the fulfillment of the inequality $n > 1$ is required for the descending branch to occur on the $h_{\text{t}}(Q_T)$ plot. Thus, under the conditions given above, the ascending branch of the characteristic $H(Q_T)$ in the nonviscous-gas flow appears because of the descending branch of $h_{\text{t}}(Q_T)$. An analogous result is obtained in analyzing the head loss equal to the difference of the total pressures and caused by the heat supply in the flow of an ideal gas.

The lift pressure $A(Q_T)$ is involved in the head characteristic $H(Q_T)$ and contributes to the generation of its ascending branch, if the function $\psi(Q_T)$ in the device (Fig. 1) increases. This is the second reason why self-oscillations are excited in vertical primary furnaces; this reason is the most substantial for blast-furnace stoves.

The increase in the hydraulic resistance of the throttle in the device (Fig. 1) contributes to the fact that the temperature in the flow will grow with heat load more appreciably than velocity. Under this condition, the Reynolds number decreases with increase in the flow rate. The $h_{\text{leng}}(Q_T)$ descending branch of hydraulic loss along the length is formed in the process of turbulent-to-laminar transition because of the reduction in the loss, which is the third mechanism of thermoacoustic oscillations of a "singing" flame. Further increase in the flow rate will contribute to the growth in the hydraulic loss in accordance with the laminar regime of motion, if it is preserved.

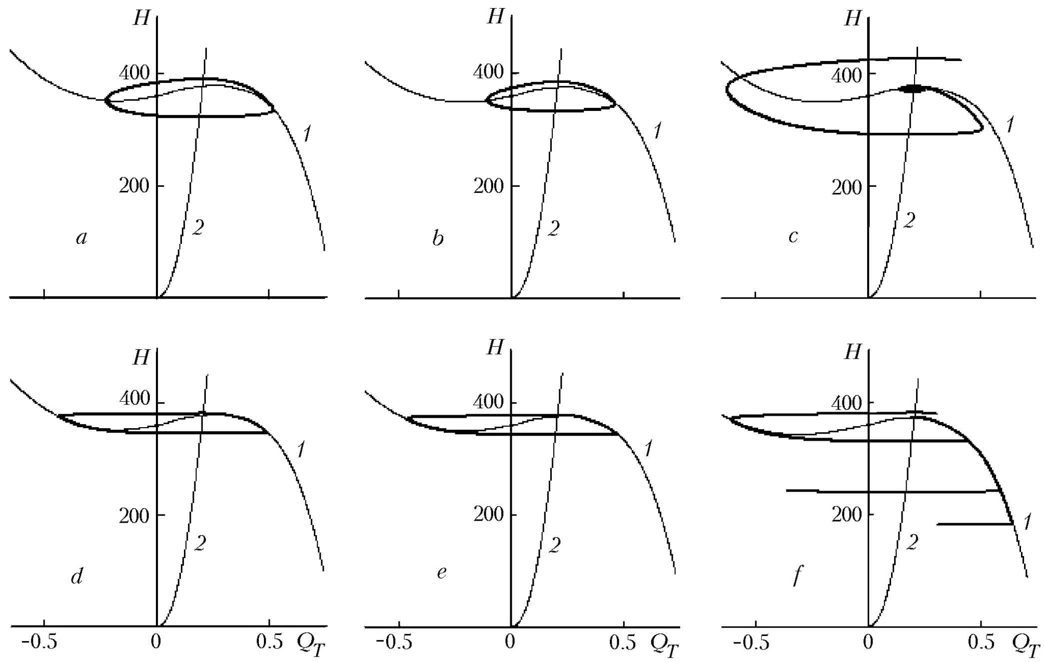

Fig. 3. Disappearance of the limit cycles of relaxation oscillations with increase in the values of the coefficient of vortex resistance: (a and d) k_v = 0, b and e) 50, and c and f) 150 for different Z values: a–c) Z = 79.63 and d–f) 17.8 Pa·sec/m^3. H, Pa; Q_T, m^3/sec.

Relaxation Self-Oscillations of Vibrational Combustion and Methods for Reducing Their Amplitude. Self-oscillations are harmonic, when the values of the wave resistance Z are high. Their amplitudes increase with manifestation of the action of phenomenological retardation (Fig. 2). As the wave resistance decreases, the oscillations become relaxation ones, whereas under the condition that $Z < Z^*$ their amplitudes are independent of further decrease in the quantity Z and on the change in the flow rate Q_T in the system, as has already been noted. Upon the transition to relaxation oscillations, the dependence of the amplitude on the manifestation of the retardation τ becomes weaker. The limit cycle of constant-amplitude relaxation self-oscillations or the cycle close to it (Fig. 2) is independent of the retardation τ.

Thus, for the vibrational-combustion mechanism due to the phenomenological retardation, the oscillation amplitude remains constant under the conditions in question. In such a regime, self-oscillations are mainly determined by the character of heat-to-head conversion, i.e., by the head characteristic $H(Q_T)$. The limit cycle increases with heat-flux power [8].

An efficient method for controlling thermoacoustic self-oscillations of a "singing" flame is the introduction of the active vortex resistance $h_v = k_v Q_T^2$, in which k_v = var, into the primary furnace. In this case we have a controlled change in the characteristic $H(Q_T)$, which enables us to control self-oscillations. Figure 3a–c shows the character of deformation and the disappearance of the limit cycle of variable-amplitude relaxation oscillations with increase in the k_v value determining vortex loss. Decrease in the wave resistance Z in the primary furnace contributes to the formation of a limit cycle of constant amplitudes, which is independent of it and, for a certain value of the increase in the wave resistance, degenerates into a stable focus (Fig. 3d–f) without being changed.

The same limit cycle of constant-amplitude oscillations is preserved in the case of series connection of a passive oscillatory circuit of dynamic damping, through which straight-through motion is carried out.

The second efficient method for controlling and neutralizing constant-amplitude oscillations is the transformation of the primary furnace to a "honeycomb" one (Fig. 4), in which the acoustic masses L_{aj} of each individual primary furnace are increased, and the acoustic flexibility of oscillatory circuits may be variable. In this structure, a constant limit cycle becomes variable due to the increase in the wave resistance $Z = \sqrt{L_{aj}/C_{a,var}}$ in individual primary

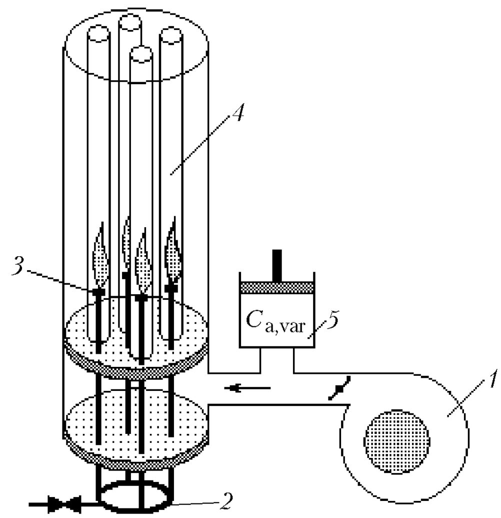

Fig. 4. "Honeycomb" primary furnace with individual burners and a variable acoustic flexibility $C_{a,var}$: 1) fan; 2) gas collector; 3) atomizers; 4) honeycomb primary furnaces; 5) additional mass accumulator of the oscillatory circuit.

furnaces [14], and it is brought closer to the cycle of nearly harmonic oscillations by changing the quantity $C_{a,var}$, if it seems possible for the corresponding structure.

The character of motions in devices of a "honeycomb" primary furnace corresponds to limit cycles dependent on the wave resistance Z and determined by the condition $Z < Z^*$ [11] (Fig. 2), whereas the harmonic oscillations are easily controlled.

CONCLUSIONS

1. When the harmonic oscillations of vibrational combustion are transformed to relaxation ones, the influence of the phenomenological combustion retardation on the oscillation amplitude becomes weaker. The formation of a limit cycle of constant-amplitude relaxation oscillations or of that close to it totally eliminates such influence and the value of the amplitude for these conditions is mainly determined by the intensity of conversion of heat to the flow head, growing with heat-supply power.

2. Relaxation self-oscillations corresponding to the limit cycle of constant-amplitude oscillations do not change the value of the amplitude in dynamic damping.

3. Control of such oscillations is difficult; it involves the action on the mechanisms of their excitation and sustaining by the use of which one can decrease the values of the intensity dH/dQ_T of the ascending branch of the head characteristic $H(Q_T)$.

4. Increase in the wave resistance Z of the oscillatory circuit transforms the limit cycle of constant-amplitude oscillations to a variable cycle, which enables one to control the oscillations; a certain influence of the mechanisms of phenomenological retardation on the oscillation amplitude manifests itself.

NOTATION

a_T, velocity of propagation of sound in a heated medium, m/sec; $A(Q_TW)$, lift pressure, Pa; c, flow velocity, m/sec; C_a, acoustic flexibility, m^3/Pa; c_v, specific heat at constant volume, J/(kg·K); g, free-fall acceleration, m/sec^2; H, flow head, Pa; $H(Q_T)$, head characteristic of heat supply, Pa; $H(Q_TW)$, head characteristic of the Riecke tube, Pa; $H_f(Q_T)$, head characteristic of the fan, Pa; $h_n(Q_T)$, characteristic of the network connected to the oscillatory circuit, Pa; h_v, hydraulic loss due to vortex formation, Pa; $h_{leng}(Q_T)$, hydraulic loss along the channel length, Pa; $h_t(Q_T)$, head loss due to heat supply, Pa; k, adiabatic exponent; k_v, proportionality factor in the dependence for vortex loss, Pa/(m^3/sec)^2; k_n, proportionality factor in the dependence for $h_n(Q_T)$, Pa/(m^3/sec)^2; L_a, acoustic mass, Pa·sec^2/m^3; L_{aj}, acoustic masses of individual primary furnaces, Pa·sec^2/m^3; n, polytropic index; p_{bl}, pressure at the fan outlet, Pa; p_{furn}, pressure in the primary furnace, Pa; P, pressure difference, Pa; Q_T, volumetric rate of flow going out of the primary furnace, m^3/sec; $Q_{in}(t-\tau)$, volumetric rate of flow entering the primary furnace, m^3/sec; q, supplied heat, J/kg; S, cross-sectional area of the flow, m^2; T, absolute temperature, K; t, time, sec; u, internal energy, J/kg; W, power of the stove electric coil, W; Z, wave resistance of the primary furnace, Pa·sec/m^3; Z^*, wave resistance corresponding to the formation of a limit cycle of constant-amplitude oscillations, Pa·sec/m^3; Z_{leng}, length of the primary furnace, m; ρ, gas density in the flow, kg/m^3; τ, retardation, sec; φ(P), inversion of the function determining the characteristic of the network, m^3/sec; ψ(Q_T), function determining the temperature as a function of the flow rate, K. Subscripts: a, acoustic; v, vortex; v, process at constant volume; var, variable; f, fan; out, outlet from the oscillatory circuit; in, inlet to the circuit; furn, primary furnace; bl, blowing at the fan outlet; n, characteristic of the network connected to the oscillatory circuit; leng, dependence on length; t, thermal resistance; T, heated medium; 0, initial value of the parameter.

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